Numerical model of radial wind velocity in case of Gaussian approximation of range weighting function

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Abstract

In this paper range weighting function for a Gaussian approximation is proposed. Using this approximation a standard deviation of radial wind velocity is calculated depending on the Doppler lidar parameters and state of atmospheric turbulence during the day. The results of numerical simulation of standard deviation show that the difference between approximation and exact formulas is 20%.

Keywords: radial wind velocity, Gaussian approximation, range weighting function, turbulent atmosphere

Introduction

It is shown in [1] that the Doppler lidar signal is a nonstationary non-Gaussian random process with Gaussian conditional statistical characteristics. In [2] using the statistical analysis of the Doppler lidar signal in the turbulent atmosphere and the perturbation theory the non-Gaussian model of estimation of the radial wind velocity in turbulence atmosphere was proposed. This model is difficult for computation; therefore it is necessity to obtain approximation formulas. In this paper using a Gaussian approximation of range weighting function we obtain an approximation formula of the measurement statistical uncertainty of mean radial wind velocity and also we compare it with the exact formula.

Non-Gaussian model of radial wind velocity

It is shown in [2] that for a homogeneous and isotropic turbulence the variance $\sigma_{u_r}^2 = \left\langle \left(\hat{u}_r - u_r\right)^2 \right\rangle$, which characterize measurement statistical uncertainty of mean radial wind velocity is

$$\sigma_{u_r}^2 = \int dR_1 dR_2 w(R_1) w(R_2) K_r(R_1, R_2) + \frac{\lambda \sqrt{4\left\langle \Delta u_r'^2 \right\rangle - 3\left\langle \Delta \tilde{u}_r'^2 \right\rangle}}{8\sqrt{\pi} M_s T_s} + \frac{2\left\langle \Delta u_r'^2 \right\rangle}{\mathrm{SNR} M_s} + \frac{\lambda^2}{48 \mathrm{SNR}^2 M_s T_s^2} + \delta, \tag{1}$$

$$\left\langle \Delta u_r'^2 \right\rangle = \frac{1}{8\tau_0^2 k^2} + \frac{1}{2}\int_{-\infty}^{\infty} dR_1 dR_2 w(R_1) w(R_2) D_r(R_1, R_2), \tag{2}$$

$$\left\langle \Delta \tilde{u}_r'^2 \right\rangle = \frac{1}{8k^2\tau_0^2} + \frac{1}{2}\int_{-\infty}^{\infty} dR_1 dR_2 \left| p_0\left(2\frac{R_1}{c}\right)\right|^2 \left| p_0\left(2\frac{R_2}{c}\right)\right|^2 D_r(R_1, R_2) \tag{3}$$

and

$$\delta = \frac{1}{16k^2 M_s T_s \tau_0} \int_{-\infty}^{\infty} dr \frac{(r^2 - 1)\times \exp\left(-\frac{r^2}{2}\right)}{\sqrt{1 + 4\tau_0^2 k^2 D_r(d_{1/2} r)}}, \tag{4}$$

where $K_r(R_1,R_2)=\langle u_r^{'}(R_1,\phi,\theta)u_r^{'}(R_2,\phi,\theta)\rangle$ and $D_r(R_1,R_2)=D_r(R_1-R_2)=\left\langle\left[u_r^{'}(R_1,\phi,\theta)-u_r^{'}(R_2,\phi,\theta)\right]^2\right\rangle$ are the correlation and structure functions, respectively; $\hat{u}_r$, u_r, and $u_r^{'}$ are estimation, mean value, and fluctuations of radial wind velocity, respectively; $w(R)=\frac{1}{M_s}\sum_{k=-m}^{k=m}\left|p_0\left(t_0+kT_s-2\frac{R}{c}\right)\right|^2$ is a range weighting function, $|p_0(t)|^2=\frac{1}{\sqrt{\pi\tau_0^2}}\exp\left\{-\frac{t^2}{\tau_0^2}\right\}$, $2\tau_0$ is pulse duration; T_s and M_s are time and number of samples, and SNR is signal to nose ratio [2].

Gaussian approximation of range weighting function

The range weighting function can be written as

$$w(R)=\frac{1}{M_s}\sum_{k=-m}^{k=m}\left|p_0\left(t_0+kT_s-2\frac{R}{c}\right)\right|^2=\int_{-\infty}^{\infty}\mathrm{T}(R)\exp\left\{-\frac{(R-r)^2}{d_{1/2}^2}\right\}dR \tag{5}$$

For the function $T(t)$ we use the Gaussian approximation $\mathrm{T}(R)=T_0\exp\left\{-\frac{R^2}{R_0^2}\right\}$. For this approximation the range weighting function has the form

$$w(r)=\frac{1}{\sqrt{\pi}d_{1/2eff}}\exp\left\{-\frac{r^2}{d_{1/2eff}^2}\right\}, \tag{6}$$

$$d_{1/2eff}^2=\Delta p_{1/2}^2+d_{1/2}^2, \tag{7}$$

where $\Delta p_{1/2}=\frac{M_sT_sc}{4}$, and $d_{1/2}=\frac{c\tau_0}{2}$.

In [3] an approximation formula of range resolution ΔR is proposed for investigation of the effects of wind turbulence on pulsed coherent Doppler lidar performance. This approximation formula has the form $\Delta R=\Delta p+\Delta r$, where $\Delta p=\frac{M_sT_sc}{2}$ is the range gate and $\Delta r=\sqrt{\ln 2}c\tau_0$ is the dimensions of the aerosol region illuminated by the pulse at a given time t. Using the results of [3] we can interpret the values of $2d_{1/2eff}$, $2\Delta p_{1/2}$, $2d_{1/2}$ as the range resolution, the range gate, and the dimensions of the aerosol region illuminated by the pulse at a given time t, respectively. In this paper the value of $2d_{1/2}$ is defined at the level of e^{-1}, but in [3] it is defined at the level of 1/2. It is clear that for $\sqrt{\ln 2}\cong 1$ our definition of the range resolution coincide with approach in [3] and $2d_{1/2eff}=\Delta R$ if $d_{1/2}^{\pm 1}\to 0$. So the Gaussian approximation is good asymptotic value of the range weighting function dimension.

Model of the turbulent atmosphere

In the case of a homogeneous and isotropic turbulence and Kolmogorov's “2/3” Law [4] the correlation and structure functions have the forms

$$K(r)=\frac{2}{3}e-\frac{1}{2}C^2\varepsilon^{2/3}r^{2/3} \tag{8}$$

$$D_r(r) = C^2\varepsilon^{2/3}r^{2/3} \tag{9}$$

where e is turbulent kinetic energy, ε - energy of dissipation, and $C^2 = 1.77$ is a coefficient. For these functions it is true [4]

$$K_r(R_1, R_2) = \frac{2}{3}e - \frac{1}{2}D_r(R_1, R_2). \tag{10}$$

Approximation formula of non-Gaussian model of radial wind velocity

For Kolmogorov's "2/3" Law and Gaussian approximation of range weighting function (see Eqs. (6)–(10)) the variance has the following form:

$$\sigma_{u_r}^2 = \frac{2}{3}e - 0.401C^2\varepsilon^{2/3}d_{1/2eff}^{2/3} + \frac{\lambda\sqrt{4\langle\Delta u_r'^2\rangle - 3\langle\Delta\tilde{u}_r'^2\rangle}}{8\sqrt{\pi}M_sT_s} + \frac{2N\langle\Delta u_r'^2\rangle}{SM_s} + \frac{N^2\lambda^2}{48S^2M_sT_s^2} + \delta \tag{11}$$

where $\langle\Delta u_r'^2\rangle = \frac{1}{8\tau_0^2k^2} + \varepsilon^{2/3}C^2 \times 0.401d_{1/2eff}^{2/3}$ and $\langle\Delta\tilde{u}_r'^2\rangle = \frac{1}{8\tau_0^2k^2} + \varepsilon^{2/3}C^2 \times 0.401d_{1/2}^{2/3}$.

It follows from Eq. (11) that the variance of radial wind velocity is simply calculated using information about the Doppler lidar, turbulent kinetic energy and energy of dissipation.

Results of numerical calculations

In this paper the statistical analysis was studied using 1D model of homogeneous planetary boundary layer and «e-l» model of turbulence. Figure 1presents the results of numerical simulation of the diurnal variations of the standard deviation $\sigma_{u_r} = \sqrt{\sigma_{u_r}^2}$. For numerical calculations we used the non-Gaussian model of the radial wind velocity estimation and the approximation formula (11). For numerical simulations we used the following Doppler lidar parameters λ=2.0218 micron, τ_0=0.12×10^{-6} s, S/N>>1, T_s=0.02×10^{-6} s, M_s=16.

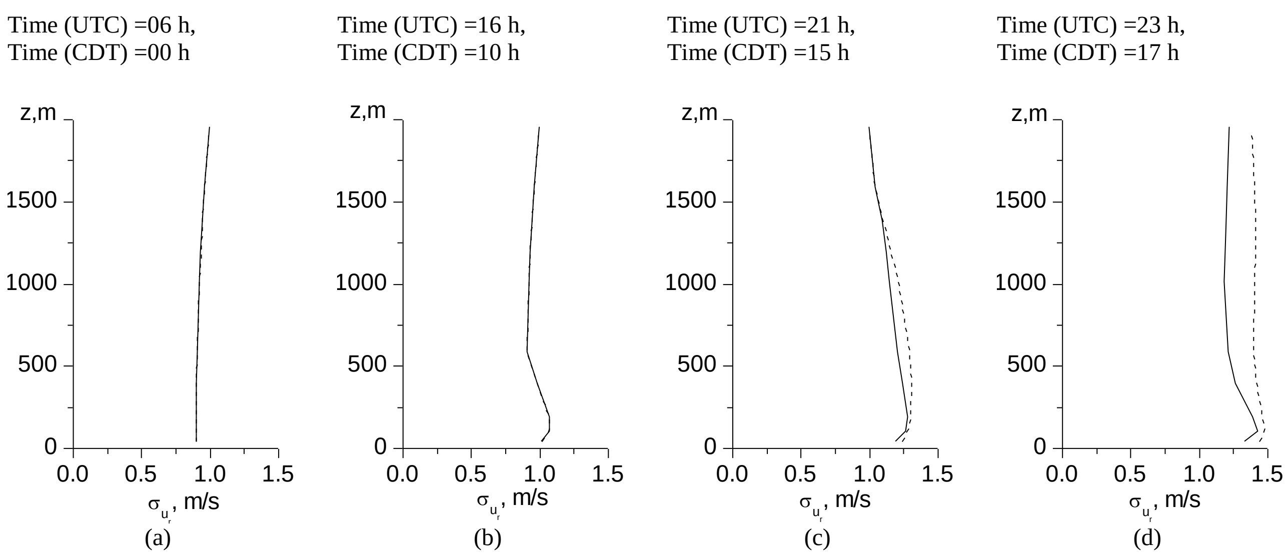

Fig. 1. The diurnal variations of the standard deviation σ_{u_r}, m/s. Solid line is exact results, dash line is approximation results

The figure 1 presents the numerical simulation results of standard deviation during the day and for M_s=16. The figures 1a,b correspond to night time and therefore it is weak turbulence. It is clear from figures 1a,b that the difference between the approximation and exact formulas is negligible. The figures 1c,d describe the standard deviation in the daytime, where the strong turbulence is observed. So it follows from figures 1c,d that the difference between the exact and approximation formulas is increased by 20%.

Conclusion

In this paper we proposed the approximation formula for the calculation of the statistical uncertainty of the measurement of mean radial wind velocity. This formula was calculated using the Gaussian approximation of range weighting function, which was described in this paper in detail. This approximation formula of range weighting function coincided with the results of paper [3] in limiting cases. The calculations using the approximation formula of standard deviation were compared with the exact results of the non-Gaussian model of radial wind velocity estimation in turbulent atmosphere. The comparison results showed that for M_s=16 the difference between the approximation and exact formulas is negligible in the case of weak turbulence. The difference between the formulas is increased by 20% for strong turbulence.

References

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